The Counting Stick: Guidance Notes

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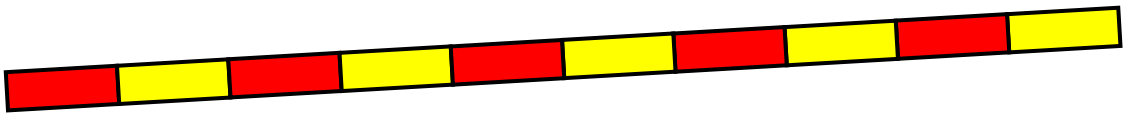
What is a counting stick?



A counting stick is a way of creating an extremely versatile example of a **marked but unlabelled number line**. It is simply part of a 'broom handle' divided into ten equal parts! It can be any length although most counting sticks are one metre long with each part 10 centimetres long. They are therefore easy to make – wrapping electrical tape in two different colours around the stick can visually provide a strong contrast between adjacent parts. Although counting sticks can be bought or metre rules used instead my recommendation has always been that you make your own in the style that best suits you!

It allows the teacher to work on a range of activities that can be modelled using an empty number line. The two ends of the stick and the points where adjacent parts meet can represent 10 equally spaced points on any number line. Children can be invited to count in steps from different starting points. The teacher can decide what the starting point should be and what step size to use.

The counting stick has a unique role to play in the teaching and learning of numeracy, particularly now that there is an increased recognition of the importance of different models and images that can be used to develop children's understanding of number and calculation.

The counting stick can be used to support work on mental calculation, the quick recall of basic number facts, to develop a fundamental understanding of fractions, decimals and percentages and their relationship to one another, and a whole lot more! It can be used when working with the whole class of groups of children. The counting stick helps children to appreciate the 'positional aspect of number', i.e. the position of one number relative to another number or numbers. This is a key skill if children are to use appropriate and efficient mental calculation strategies.

As children develop their mental calculation skills they will learn more efficient counting strategies so that they do not rely on just counting in ones or always going back to start at zero.

For example, to work out 41 − 17, rather than counting up in ones, 18, 19, 20, 21, 22, ……..40, 41 to find the difference they might count 17, 20, 40, 41 memorising the 'total jumps' as 3+20+1=24.

Examples of counting activities:

- 0, 1, 2, 3, ..., 10 and 10, 9, 8, ..., 0
- 0, 2, 4, 6, ..., 20 and 20, 18, 16, ...0
- 0, 5, 10, ..., 50 and 50, 45, 40, ..., 0
- 0, 7, 14, 21, ..., 70 and 70, 63, 56, ..., 0
- 0, ¼, ½, ¾, ... 2¼, 2½ and 2½, 2¼, ..., ½,
- 0, 0.2, 0.4, ..., 1.8. 2 and 2, 1.8, 1.6, ..., 0.2, 0

Research shows how important it is for children to count backwards as well as forwards.

and not starting at zero:

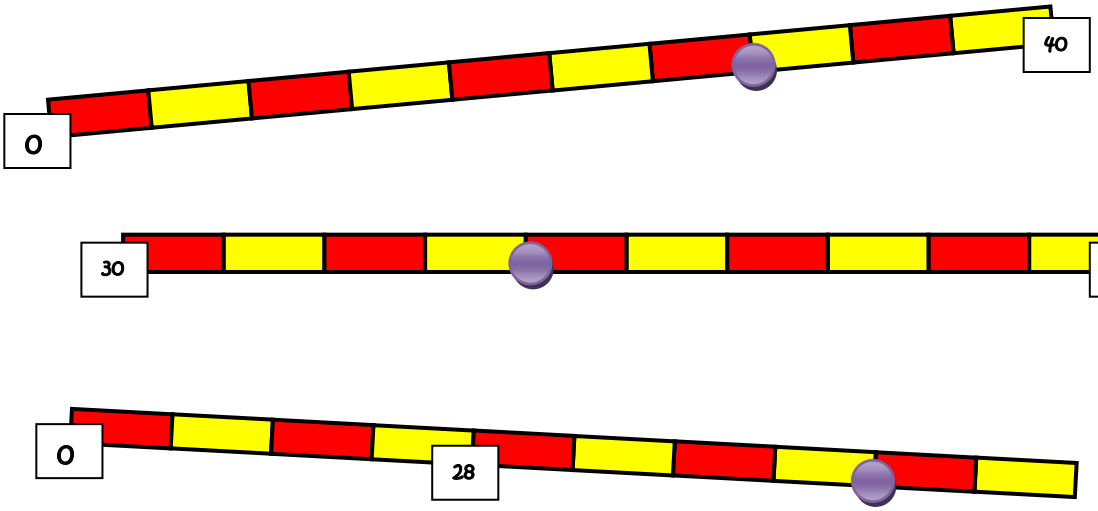
- 2, 7, 12, 17, ..., 47, 52 and 52, 47, 42, ..., 7, 2
- 97, 98, 99, ..., 105, 106 and 106, 105, 104, ..., 98, 97
- 3, 2, 1, 0, -1 ....., -5, -6 and -6, -5, -4, ..., 2, 3

Further Ideas

- Place a number on the counting stick as shown below:

Tell the pupils the size of the step, e.g. 3
What number is shown by the counter?
What numbers are at either end of the counting stick?

- Mark two numbers on the counting stick and ask pupils to find 'missing' numbers shown by counters – they will almost always first need to work out the step size.



- Try random or 'hiccup' counting. Explain to the pupils that if they can remember what the middle number represents, then this will provide them with a good clue to identifying the other numbers. 'Hop around' the counting stick pointing to different numbers at random. Regularly return to the middle point to remind the pupils that this provides them with a good reference point.

So, for example, using the counting stick is steps of 6 from 0 to 60, hiccup counting might be: 0, 30, 60, 36, 6, 12, 30, 24, 54, 60, 30, etc

- Try counting in steps from **numbers other than zero**. Many children are confident with familiar counting activities that start at zero as a step towards learning multiplication and division facts. They will certainly be less confident with activities such as counting in fives, but starting at 2, say.

- The counting stick is a fantastic resource for learning the quick recall of multiplication and division facts to 10x10. Using the number tiles (Resource Master A) can help. These can then be placed on the counting stick with 'blu-tak' so that pupils can see certain key numbers in the number sequence. As certain key facts are memorised, then the corresponding number tiles can be removed. It maybe that not all the numbers need to be placed at first, as this will encourage children to figure out those missing numbers.

  Here are some further ideas:
  Use the number tiles to help pupils first develop quick recall of the corresponding multiples. So for example learning the multiplication and division facts for 7, pupils would first need to be familiar with the sequence 0, 7, 14, 21, …. 70.
  Use the counting stick to demonstrate how some multiples can be figured out from others that they know by heart. Help the pupils to memorise the first few multiples, the 'end one' (x10) and the 'half-way one' (x5).

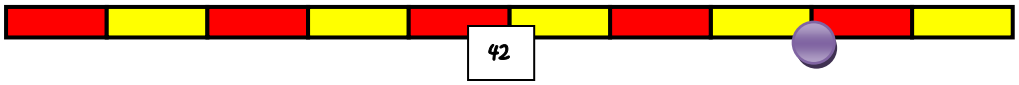
  For example:
  Show that 8x7 is double 4x7 (On the counting stick 8x7 is twice the distance from 0 that 4x7 is!)
  Alternatively show that it is also 2x7 less than 10x7. "Which is the most efficient strategy to figure out 8x7 if you did not know it by heart?"
  Show that 6x7 is (5x7) + (1x7) i.e. half-way and seven more.
  Show that 9x7 is (10x7) — (1x7) i.e. the end one take away seven, etc.
  Teachers may wish to leave some number tiles on the counting stick corresponding to certain multiples that pupils find difficult to recall quickly.

  Once these have been learnt, use the counting stick to 'chant' multiplication facts, i.e. "one times seven in seven, two times seven is fourteen, ..."
  Next move onto using the counting stick for 'random chanting' of multiplication facts. Pointing to different places at random could produce these responses: "four times seven is twenty eight; seven times seven is forty nine, ...". Don't forget that you can practice other mathematical vocabulary linked to 'multiplication', e.g. "one **multiplied** by seven is seven, two **multiplied** by seven is fourteen, ..." or "the first multiple of seven is seven, the second multiple of seven is fourteen, ..."
  Now try chanting division facts, i.e. "seven divided by seven is one, fourteen divided by seven is two, twenty one divided by seven is three, ..." Next try random chanting of division facts.
  Remember to demonstrate the link between division and multiplication and division and repeated subtraction. For example, ask pupils to think of 42÷6 as 'How many lots of six are there in forty two?' or 'How many times can I take six away from forty two?'

- These activities can be **extended to include fractions, decimals and percentages**. There are an endless number of variations and, for example, with children who are very confident with their seven times table could still try these counting activities involving 'sevens':

(a)	0, 14, 28, 42, 56, ...	(Adding 14 by doubling multiple s of 7)
(b)	15, 22, 29, 36, ...	(Adding on 7 to a non-zero number)
(c)	100, 93, 86, 79, ...	(Taking away 7 from a non-zero number)
(c)	0, 0·7, 1·4, 2·1, ...	(Adding on 0·7 starting at zero)
(e)	8400, 7700, 7000, ...	(subtracting 700 from a large number)

- Place a number in the middle and ask children what other numbers (represented by counters) could be – I use the word could because there are a range of answers and pupils would be expected to justify their responses.



Teacher: "What number could be represented by the counter?"
Pupil: "It could be 54 because there are three steps between 42 and the counter and each step could be worth 4"

- **Individual counting sticks (Resource Master B) can be easily made.**

Photocopy onto A4 or even A3 card and laminate (if possible!) An elastic band (or paper clip) can be moved along counting stick to show the position of different numbers. The 'dot' at one end represents the end from where the counting starts. Give every pupil an individual counting stick. Invite pupils to 'show me' different answers.
For example, starting at zero and counting forwards in steps of 4:
Show me 12 Show me 36 Show me 6 lots of 4
Show me half of forty Show me that 32 divided by 4 is eight
Show me where 22 would go. Be careful!!

- **Find decimals with a sum of 1**
Mark both ends of the counting stick with the numbers o and 1.

Stop at different points on the counting stick or use a small counter to mark the position of a particular point. "What number is this?" "How did you work it out?" "How many more to make 1 (or 0.1)?" Count on and check, using the divisions (or part divisions) on the counting stick to demonstrate how the answer can be worked out. Write the corresponding number sentence, e.g. 0.3+ **0.7** = 1

- **Round decimals to the nearest whole number**
  Pupils will have a clearer understanding of the idea of rounding if they can appreciate that this concept is based on the position of the number to be rounded. Therefore when rounding, say 2·6, to the nearest whole number the pupils need to know the approximate position of 2·6, i.e. it is between two and three, it is zero point six more than two but only zero point four less than three. It is therefore closer to three!
  The counting stick is an excellent resource for helping pupils develop their understanding of the positional value of numbers. The individual counting sticks are particularly effective for this activity. Similarly a number such as 2·68 could be positioned approximately on the counting stick and its distances from both 2 and 3 compared. As an extension, rounding 2·68 to one decimal place, the ends of the counting stick could be changed to 2·6 and 2·7 and then 2·68 can be placed 'easily'.

- **Recognise the equivalence of percentages, fractions and decimals**.
  Numerate pupils will recognise that percentages, fractions and decimals are clearly linked and consequently will be able to convert easily from one form to another.
  Mark the ends of the counting stick with 0 and 1.
  Count forwards and backwards in steps of 0·1, or tenths, or lots of 10%. Mark a point and invite pupils to identify its value as a fraction and a decimal and a percentage.
  'Hiccup' counting can be quite fun here. Move around the counting stick switching between fractions, decimals and percentages, calling out either Fractions! Decimals! Percentages! to indicate the change. For example: Fractions: zero, one half, three tenths, nine tenths, Percentages!! 20%, 80%, 30%, 45%, Decimals!! 0.4, 0.7, 0.5, 0.25, etc

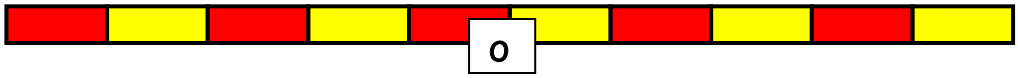
- **Find simple percentages of small whole number quantities**.
  Often simple percentages can be figured out by first finding 10% or one tenth. Then 5% will be ½ of 10%, 40% will be 4 x 10% etc. So to find simple percentages of, say 60, mark the ends of the counting stick with 0 and 60.

  Tell the pupils that the counting stick will represent 60, so we can say that 100% = 60. Now point to the first point. "What does this represent?" Answer: 10% of 60 = 6. Now point to other places on the counting stick and ask pupils to tell you what this point represents. For example, this could produce the following responses:
  40% of 60 = 24, 70% of 60 = 42, 35% of 60 = 21, etc
  Demonstrate other relationships such as the answer to 35% is half the answer to 70%, that 60% add 40% will equal 60 (or 100%), etc.

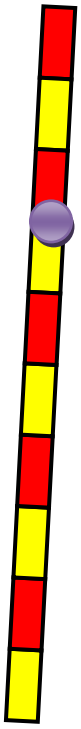
- **Use fraction as an operator to find fraction of numbers or quantities.**
  To find one tenth, two tenths, etc of a number or quantity just repeat the last activity using fractions rather than percentages.

  The same counting stick could also be used to find one fifth, two fifths, etc by taking every other point on the counting stick. The number tiles marked in fifths placed on the counting stick between zero and one would probably make the demonstration more effective.

- **Order, add and subtract positive and negative numbers.**
  Mark zero in the middle of the counting stick.



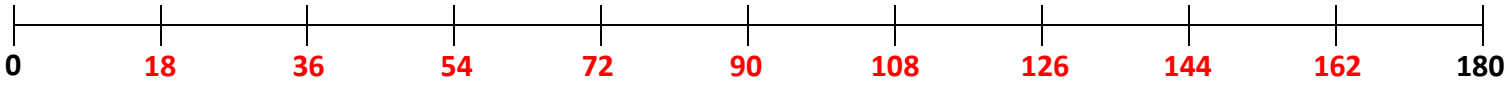
Count forwards and backwards in steps of different sizes through zero. Position positive and negative numbers on the counting stick. Use this to help pupils order a set of positive and negative numbers. Addition can be demonstrated by counting on (adding on a positive number) or counting back (adding on a negative number).



- **Read and interpret scales.**
  The counting stick represents an empty number line or scale.
  Sometimes it will be more appropriate to hold the counting stick vertically so that it represents, say the scale on a measuring cylinder.
  Mark a point. "How many millilitres are there in the cylinder?"
  "How many more millilitres would I need to pour into the cylinder before it is full?"
  "Where would 520ml be on the scale?"
  Individual counting sticks could be used.
  For example:"Show me 740ml".
  "Show me something more that ¾ full", Etc.

- Using marked, unlabelled number lines (Resource Master C) to support the teaching of fractions, decimals, percentages, ratio and proportion

  Example 1: Using scales marked from 0 to 180 and labelling each division



Examples of possible questions that could be asked – pupils can 'read' the answers from their number line:

What is 20% of 180? (**36**)
What is four tenths of 180? (**72**)
What is 0.7 of 280? (**126**)
What is 15% of 180? (18 + 9 = **27**)
What is 0.35 of 180? (54 + 9 = **63**)
What is 60% of 360? (2 x 108 = **216**)
What is 0.45 of 540? ({72 + 9 = 81} x 3 = **243**)

What percentage of 180 is 72? (**40%**)
What fraction of 180 is 135? (**¾**)
Increase 180 by 30% (180 + 54 = **234**)
Decrease 180 by 10% (180 – 18 = **162**)
Simplify the ratio 54:126 (**3:7**)
Simplify the ratio 72:108 (**4:6** or **2:3**)

Resource Master A: Examples of different sets of tiles

0	1	2	3	4
5	6	7	8	9
10	20	30	40	50
60	70	80	90	100

2	4	6	8	10
12	14	16	18	20
3	6	9	12	15
18	21	24	27	30

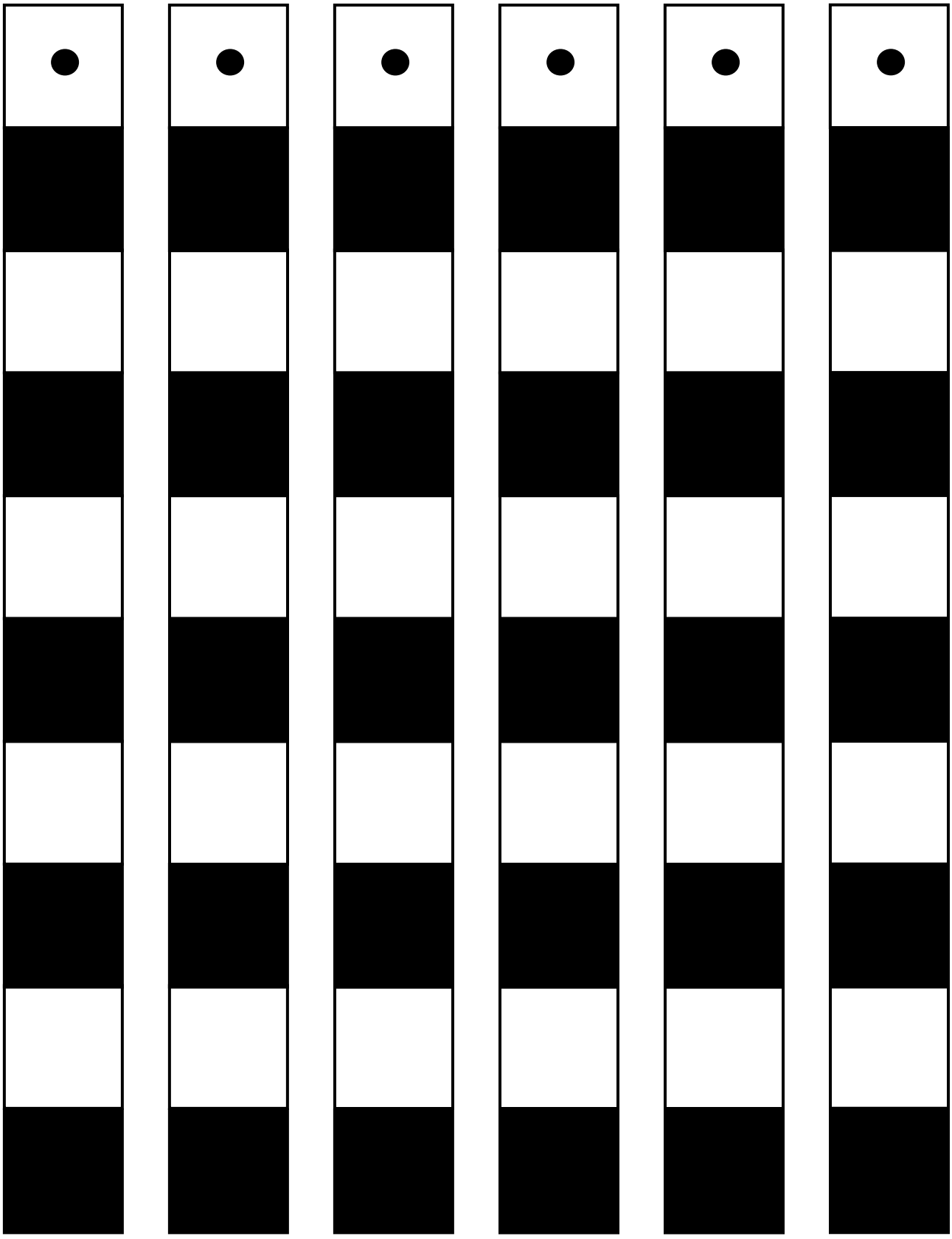
4	8	12	16	20
24	28	32	36	40
5	10	15	20	25
30	35	40	45	50

6	12	18	24	30
36	42	48	54	60
7	14	21	28	35
42	49	56	63	70

8	16	24	32	40
48	56	64	72	80
9	18	27	36	45
54	63	72	81	90

0.1	0.2	0.3	0.4	0.5
0.6	0.7	0.8	0.9	1.0
¼	½	¾	1	1¼
1½	1¾	2	2¼	2½

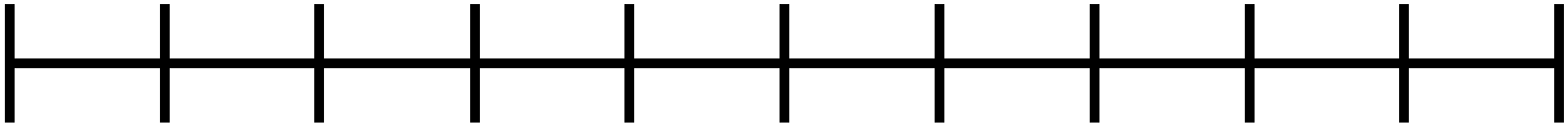
Resource Master B: Pupil
Individual Counting Sticks



Resource Master C: Pupil Individual marked but unlabelled number lines

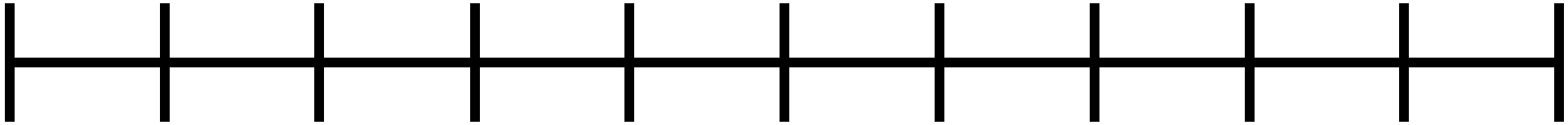
Pupil Number Line

Name: .....



Pupil Number Line

Name: .....



Pupil Number Line

Name: .....