Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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MATHEMATICS **0580/42**

Paper 4 (Extended) **February/March 2016**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator
Tracing paper (optional)
Geometrical instruments

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 1/Level 2 Certificate.

This document consists of **16** printed pages.

DC (ST/SW) 112494/2

CAMBRIDGE International Examinations

[Turn over

1 Aasha, Biren and Cemal share \$640 in the ratio 8 : 15 : 9.

(a) Show that Aasha receives \$160.

Total parts = 8 + 15 + 9 = 32

Aasha = $\frac{8}{32} \times 640$

= 160 \$ Hence proved.

[1]

(b) Calculate the amount that Biren and Cemal receive.

Biren = $\frac{15}{32} \times 640 = 300$ \$

Cemal = $\frac{9}{32} \times 640 = 180$ \$

Biren \$ 300

Cemal \$ 180 [2]

(c) Aasha uses her \$160 to buy some books.
Each book costs \$15.25 .

Find the greatest number of books that she can buy.

No of books = $\frac{\text{Total}}{\text{Cost}} = \frac{160}{15.25} = 10.49$

Greatest no of books = 10

10 [2]

(d) Biren spends $\frac{3}{8}$ of his share on clothes and $\frac{1}{3}$ of his share on a computer.

Find the fraction of his share that he has left.
Write your fraction in its lowest terms.

Fraction left = $1 - \frac{3}{8} - \frac{1}{3}$

$= \frac{24 - 3\times3 - 8}{24}$

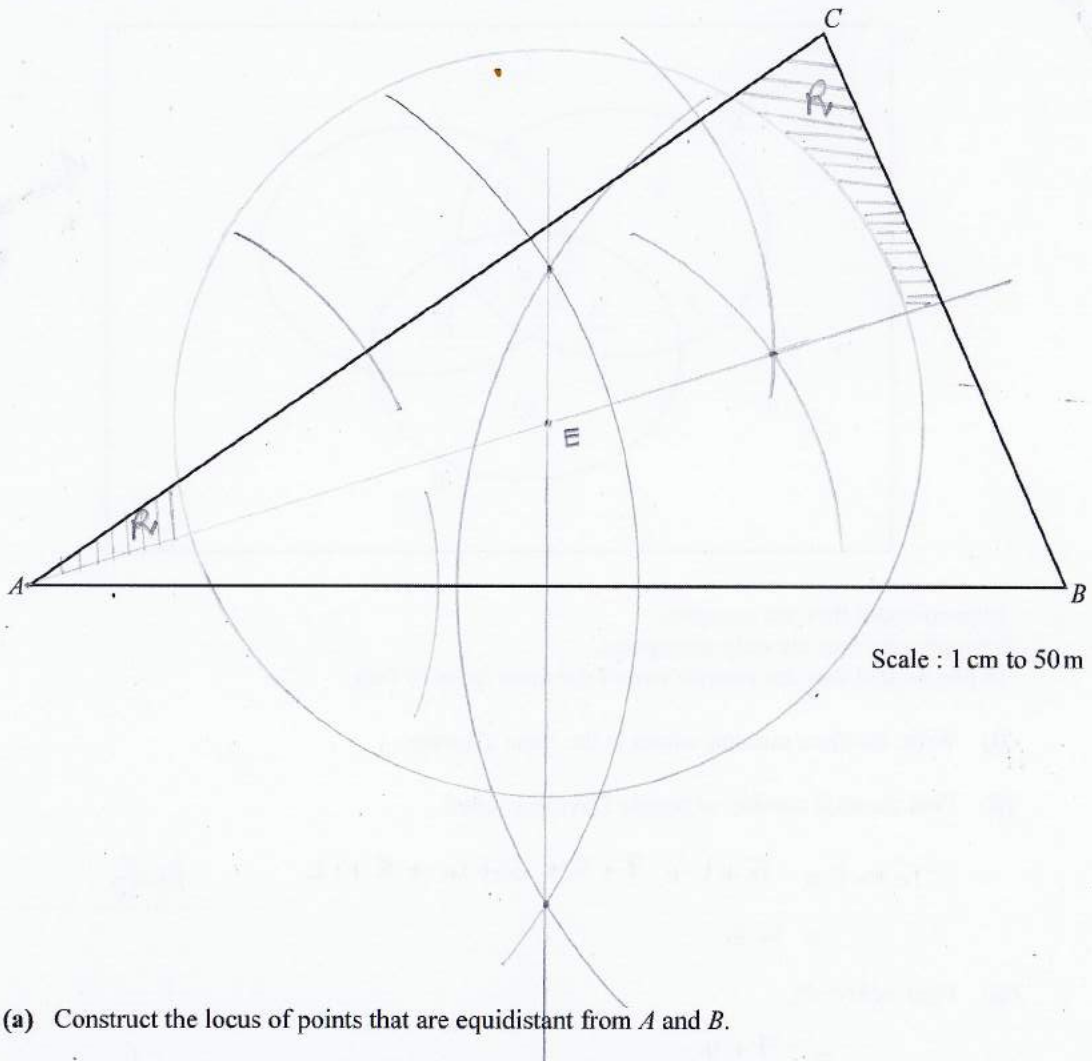
$= \frac{24 - 9 - 8}{24}$

$= \frac{7}{24}$

$\frac{7}{24}$ [3]

2 **In this question use a ruler and compasses only.**
Show all your construction arcs.

The diagram shows a triangular field ABC.
The scale is 1 centimetre represents 50 metres.



(a) Construct the locus of points that are equidistant from A and B. [2]

(b) Construct the locus of points that are equidistant from the lines AB and AC. [2]

(c) The two loci intersect at the point E.

Construct the locus of points that are 250 m from E. 250m = 5cm [2]

(d) Shade any region inside the field ABC that is

- more than 250 m from E

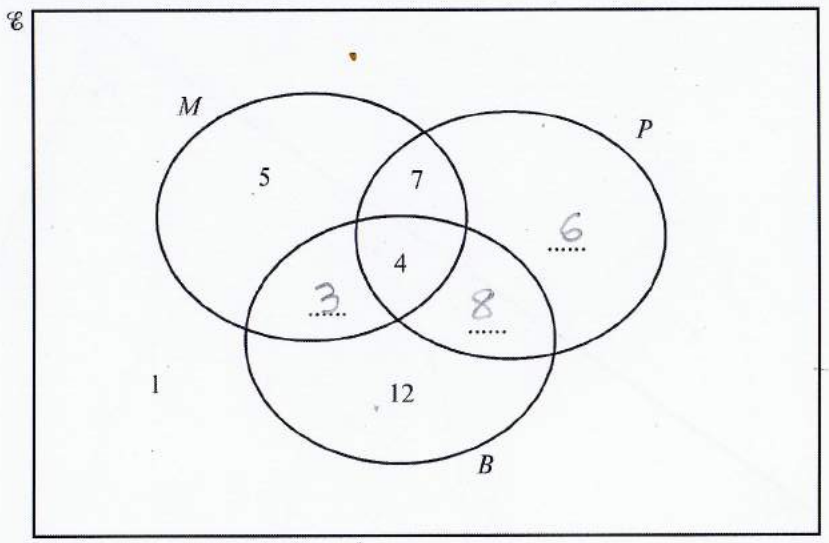
and

- closer to AC than to AB. [2]

3 **(a)** Davinder asked some people if they ate mangoes, pineapples or bananas last week.

M = { people who ate mangoes }
P = { people who ate pineapples }
B = { people who ate bananas }

The Venn diagram shows some of the information.



19 people said they ate mangoes.
6 people said they ate **only** pineapples.
18 people said they ate **exactly two** of the three types of fruit.

(i) Write the three missing values in the Venn diagram. [3]

(ii) Find the total number of people Davinder asked.

Total = 5+1+7+3+6+4+8+12
= 46

46 [1]

(iii) Find $n(M \cap P)$.

= 7+4
= 11

11 [1]

(iv) One person is chosen at random from the people who ate mangoes.

Write down the probability that this person also ate bananas.

$P(M \text{ and } B) = \frac{7}{19}$

$\frac{7}{19}$ [2]

(b) Davinder draws a speed-time graph for his bus journey to the market.

Find

(i) the acceleration of the bus during the first 200 seconds,

Two points on the line OB (0, 0) (200, 9)

Acceleration = Gradient

$= \frac{9-0}{200-0} = \frac{9}{200}$ m/s^2 $\frac{9}{200}$ (or) 0.045

..... m/s^2 [1]

(ii) the total distance travelled by the bus,

Total distance = Enclosed area of Trapezium

$= \frac{(a+b) \times h}{2}$

$= \frac{(900+1500) \times 9}{2}$

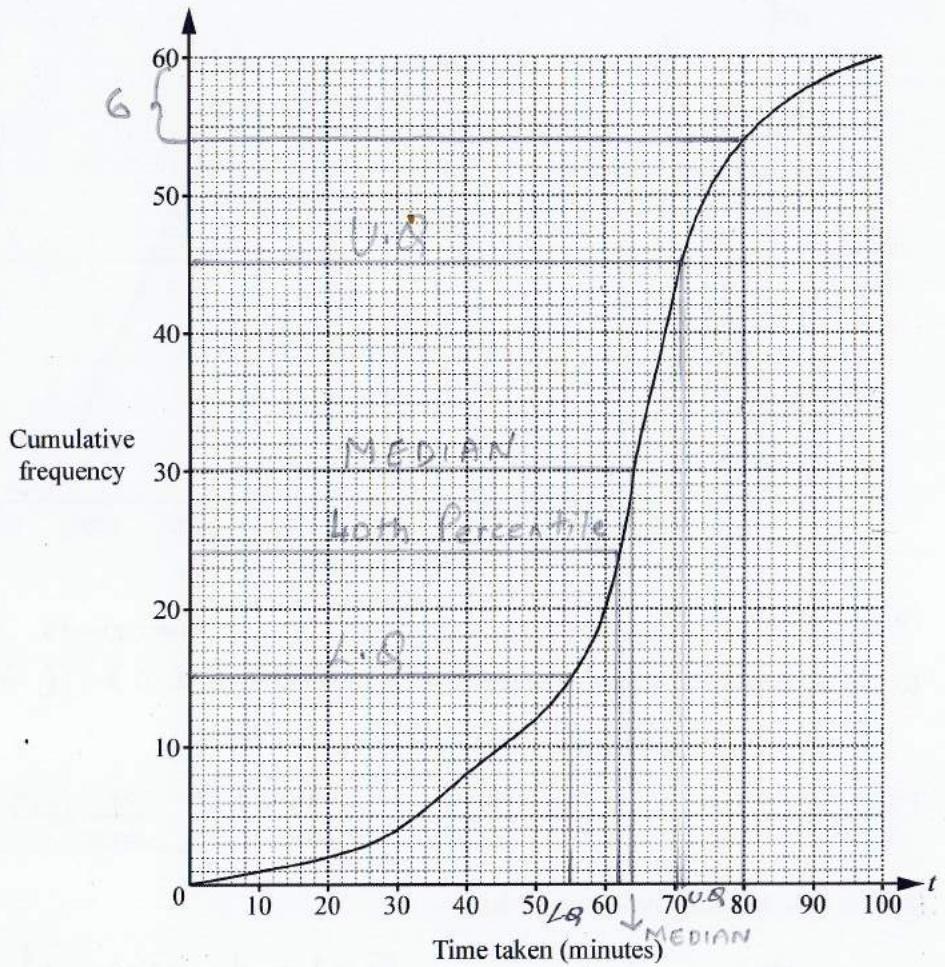
$= 10800$ m 10800 ..... m [3]

(iii) the average speed of the bus for the whole journey.

Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{10800}{1500} = 7.2$ m/s

7.2 ..... m/s [1]

4 The cumulative frequency diagram shows information about the time taken, t minutes, by 60 students to complete a test.



(a) Find

(i) the median,

$\frac{60}{2} = 30 \rightarrow 64$

64 ..... min [1]

(ii) the inter-quartile range,

UQ = 71

LQ = 55

Inter-quartile range = UQ − LQ = 71 − 55 = 16

16 to 16.5 ..... min [2]

(iii) the 40th percentile,

$\frac{40}{100} \times 60 = 24 \rightarrow 62$

62 ..... min [2]

(iv) the number of students who took more than 80 minutes to complete the test.

60 − 54 = 6

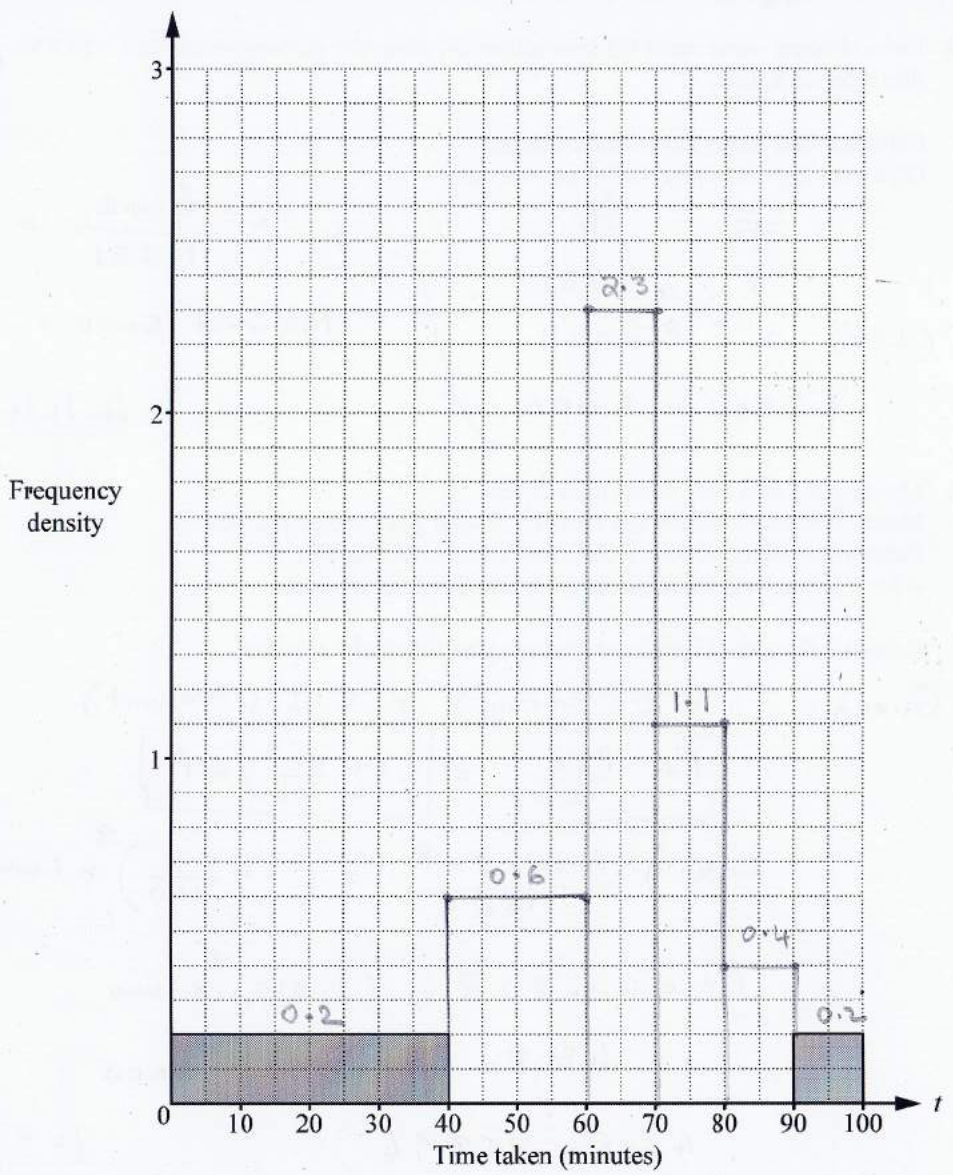
6 ..... [2]

(b) Use the cumulative frequency diagram to complete the frequency table below.

Time taken (t minutes)	$0 < t \leqslant 40$	$40 < t \leqslant 60$	$60 < t \leqslant 70$	$70 < t \leqslant 80$	$80 < t \leqslant 90$	$90 < t \leqslant 100$
Frequency	8	12	23	11	4	2

From page 6 graph: (20 − 8) (43 − 20) (54 − 43) (58 − 54) (60 − 58) [3]

(c) On the grid below, complete the histogram to show the information in the table in **part (b)**.



[4]

Frequency Density : $\frac{8}{40}$ $\frac{12}{20}$ $\frac{23}{10}$ $\frac{11}{10}$ $\frac{4}{10}$ $\frac{2}{10}$

$\left[\frac{\text{Frequency}}{\text{class width}}\right]$ 0.2 0.6 2.3 1.1 0.4 0.2

[Turn over

5 **(a)** Meena sells her car for \$6000.
This is a loss of 4% on the price she paid.

Calculate the price Meena paid for the car.

\$ %
6000 96 ← (100 − 4)
(Let) x 100

96x = 600000

$x = \frac{600000}{96}$

$x = 6250$ \$

\$ 6250 [3]

(b) Eisha changes some euros (€) into dollars (\$) when the exchange rate is €1 = \$1.351 .
She receives \$6000.

Calculate how many euros Eisha changes.
Give your answer correct to the nearest euro.

€ \$
1 1.351
(Let) x 6000

1.351x = 6000

$x = \frac{6000}{1.351} = 4441.2$

Nearest Euro = 4441

€ 4441 [3]

(c) Meena and Eisha both invest their \$6000.
Meena invests her \$6000 at a rate of 1.5% per year compound interest.
Eisha invests her \$6000 in a bank that pays simple interest.
After 8 years, their investments are worth the same amount.

Calculate the rate of simple interest per year that Eisha received.

Given: S.I (Amount) = C.I (Amount)

$P + \frac{PTR}{100} = \left(1 + \frac{R}{100}\right)^T \times P$

$6000 + \frac{6000 \times 8 \times R}{100} = \left(1 + \frac{1.5}{100}\right)^8 \times 6000$

$6000 + 480R = (1.015)^8 \times 6000$

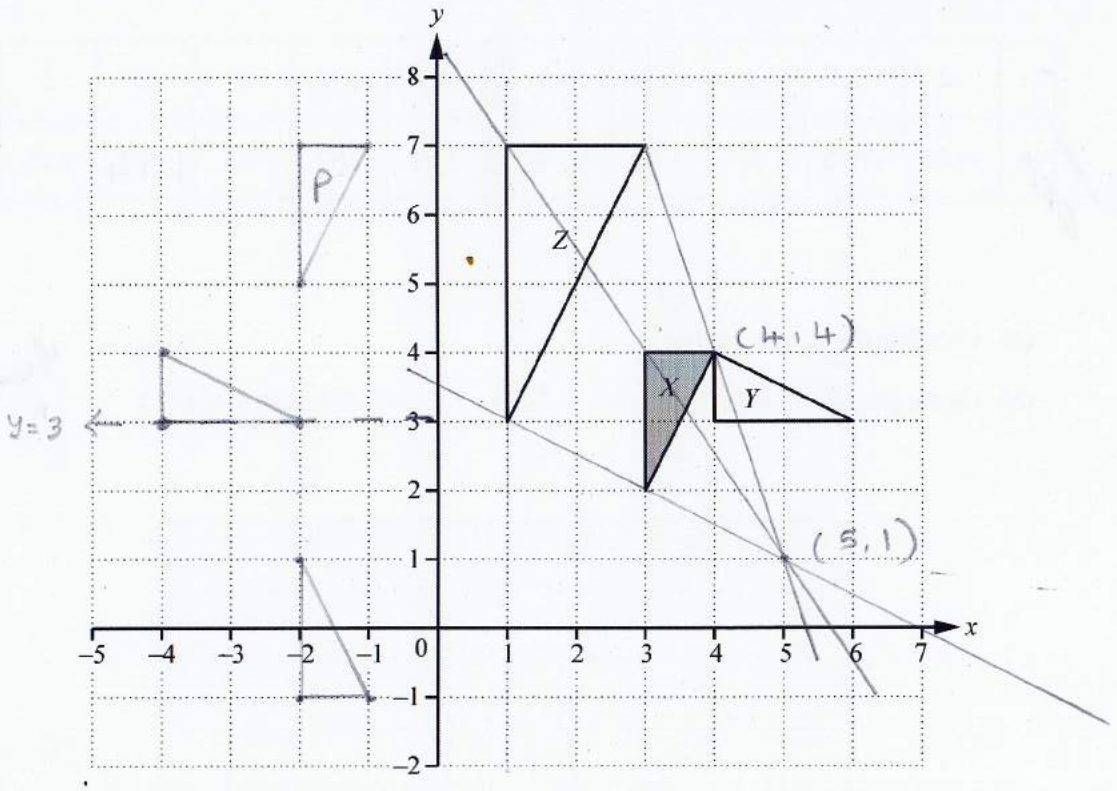
$480R = 6758.96 - 6000$

$480R = 758.96$

$R = \frac{758.96}{480} = 1.58\ \%$

1.58 % [5]

6



(a) Describe fully the **single** transformation that maps

(i) triangle X onto triangle Y,

Rotation 90° anticlockwise about (4, 4)

.....[3]

(ii) triangle X onto triangle Z.

Enlargement, scale factor 2, centre (5, 1)

.....[3]

(b) (i) Draw the image of triangle X after a translation by the vector $\begin{pmatrix} -5 \\ 3 \end{pmatrix}$.

Label this triangle P. [2]

(ii) Draw the reflection of triangle P in the line $y = 3$. [2]

(c) Draw the image of triangle X after the transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [3]

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 & 3 & 4 \\ 2 & 4 & 4 \end{pmatrix} = \begin{pmatrix} -2 & -4 & -4 \\ 3 & 3 & 4 \end{pmatrix}$$

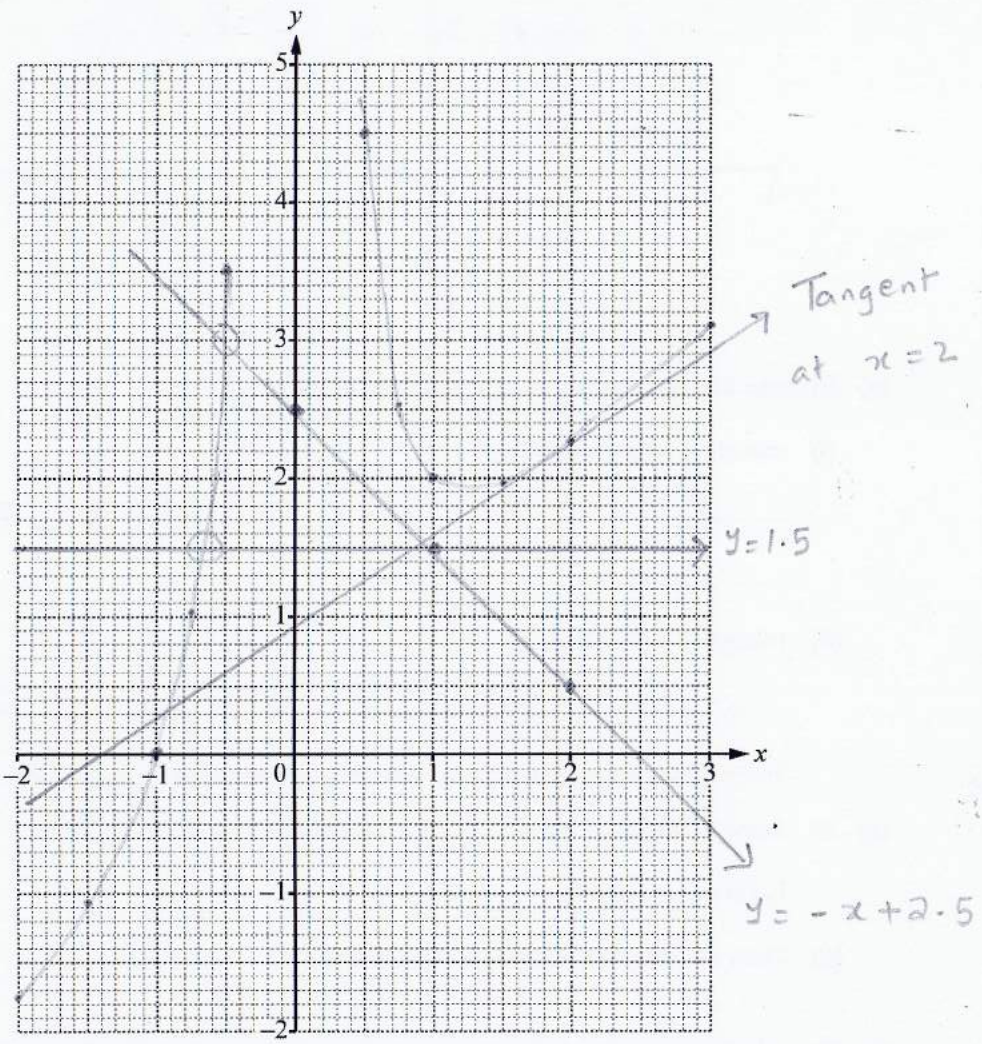
Rotation 90° anticlockwise about (0, 0).

7 The table shows some values of $y = x + \frac{1}{x^2}$, $x \neq 0$.

x	−2	−1.5	−1	−0.75	−0.5		0.5	0.75	1	1.5	2	3
y	−1.75	−1.06	0	1.03	3.5		4.50	2.53	2	1.94	2.25	3.11

(a) Complete the table of values. [3]

(b) On the grid, draw the graph of $y = x + \frac{1}{x^2}$ for $-2 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 3$.



[5]

(c) Use your graph to solve the equation $x + \frac{1}{x^2} = 1.5$.

$x =$ −0.7 to −0.6 [1]

(d) The line $y = ax + b$ can be drawn on the grid to solve the equation $\frac{1}{x^2} = 2.5 - 2x$.

(i) Find the value of a and the value of b.

$\frac{1}{x^2} = 2.5 - 2x$

Add 'x' both sides

$x + \frac{1}{x^2} = 2.5 - 2x + x$

$= 2.5 - x$

Compare $-x + 2.5$ with $y = ax + b$

$a = -1$; $b = 2.5$

$a =$ −1

$b =$ 2.5 [2]

(ii) Draw the line $y = ax + b$ to solve the equation $\frac{1}{x^2} = 2.5 - 2x$.

x	0	1	2	2.5
$-x + 2.5$	2.5	1.5	0.5	0

$x =$ −0.6 to −0.5 [3]

(e) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = 2$.

Consider the points on the tangent

(0, 0.9) (−1.4, 0)

Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0.9}{+1.4}$

$= 0.643$

0.5 ≤ gradient ≤ 0.85 [3]

8 **(a)** y is directly proportional to the positive square root of $(x+2)$.
When $x = 7$, $y = 9$.

Find y when $x = 23$.

Given: $y \propto \sqrt{x+2}$

$\Rightarrow \quad \frac{y_1}{y_2} = \frac{\sqrt{x_1+2}}{\sqrt{x_2+2}}$

$\frac{9}{y} = \frac{\sqrt{7+2}}{\sqrt{23+2}}$

$\frac{9}{y} = \frac{\sqrt{9}}{\sqrt{25}}$

$\frac{9}{y} = \frac{3}{5}$

$9 \times 5 = 3y$

$\frac{3}{\cancel{3}} \frac{9 \times 5}{\cancel{3}} = y$

$\therefore \quad y = 15$

$y =$ 15 [3]

(b) Simplify.

$\frac{x^2+12x+36}{x^2+4x-12}$ → Eq ① / Eq ②

Eq ①

Consider: $x^2 + 12x + 36$

$x^2 + 6x + 6x + 36$

$x(x+6) + 6(x+6)$

$(x+6)(x+6)$

Eq ②

Consider: $x^2 + 4x - 12$

$x^2 + 6x - 2x - 12$

$x(x+6) - 2(x+6)$

$(x+6)(x-2)$

Given: $\frac{x^2+12x+36}{x^2+4x-12}$

$= \frac{\cancel{(x+6)}(x+6)}{\cancel{(x+6)}(x-2)}$

$= \frac{x+6}{x-2}$

$\frac{x+6}{x-2}$ [5]

(c) $W = \sqrt{\frac{X-a}{a}}$

Make a the subject of the formula.

Squaring Both Sides.

$$W^2 = \frac{X-a}{a}$$

$$W^2 a = X - a$$

$$W^2 a + a = X$$

$$a(W^2+1) = X$$

$$a = \frac{X}{W^2+1}$$

$a =$ $\frac{X}{W^2+1}$ [5]

(d) Write as a single fraction in its simplest form.

$$\frac{x-2}{x+1} - \frac{x+3}{x-1}$$

$$= \frac{(x-2)(x-1) - (x+3)(x+1)}{(x+1)(x-1)}$$

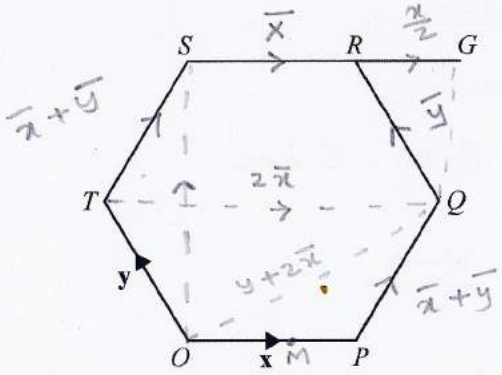
$$= \frac{\cancel{x^2} - x - 2x + 2 - \cancel{x^2} - x - 3x - 3}{x^2-1}$$

$$= \frac{-7x-1}{x^2-1}$$

$\frac{-7x-1}{(x+1)(x-1)}$ (or) $\frac{-7x-1}{x^2-1}$

..... [5]

9



NOT TO SCALE

O is the origin and $OPQRST$ is a regular hexagon.

$\overrightarrow{OP} = \mathbf{x}$ and $\overrightarrow{OT} = \mathbf{y}$.

(a) Write down, in terms of **x** and/or **y**, in its simplest form,

(i) $\overrightarrow{QR}$,

$\overrightarrow{QR}$ = $\bar{y}$ ..... [1]

(ii) $\overrightarrow{PQ}$, Consider $\triangle OPQ$

$\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$

$x + PQ = \bar{y} + 2\bar{x} \Rightarrow PQ = \bar{y} + 2\bar{x} - \bar{x} = \bar{x} + \bar{y}$

$\overrightarrow{PQ}$ = $\bar{x} + \bar{y}$ ..... [1]

(iii) the position vector of S.

$\triangle OTS$: $\bar{y} + \bar{x} + \bar{y} = \overrightarrow{OS}$

$\therefore \overrightarrow{OS} = 2\bar{y} + \bar{x}$

$\bar{x} + 2\bar{y}$ ..... [2]

(b) The line SR is extended to G so that $SR : RG = 2 : 1$.

Find $\overrightarrow{GQ}$, in terms of **x** and **y**, in its simplest form.

$\triangle RGQ$, $\overrightarrow{GQ} + \bar{y} = -\frac{\bar{x}}{2}$

$\overrightarrow{GQ} = -\frac{\bar{x}}{2} - \bar{y}$

$\overrightarrow{GQ}$ = $-\frac{\bar{x}}{2} - \bar{y}$ (or) $-\left(\frac{\bar{x}}{2} + \bar{y}\right)$ ..... [2]

(c) M is the midpoint of OP.

(i) Find $\overrightarrow{MG}$, in terms of **x** and **y**, in its simplest form.

$\triangle MQG$: $\overrightarrow{MG} + \overrightarrow{GQ} = \overrightarrow{MQ}$

$\overrightarrow{MG} + \frac{\bar{x}}{2} - \bar{y} = \frac{\bar{x}}{2} + \bar{x} + \bar{y}$

$\overrightarrow{MG} = 2\bar{x} + 2\bar{y}$

$\overrightarrow{MG}$ = $2\bar{x} + 2\bar{y}$ ..... [2]

(ii) H is a point on TQ such that $TH : HQ = 3 : 1$.

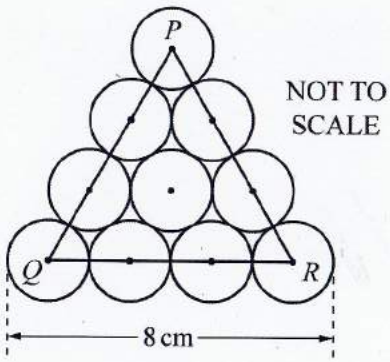
Use vectors to show that H lies on MG.

$\overrightarrow{MH} = \bar{x} + \bar{y}$ $\quad \overrightarrow{HG} = \bar{x} + \bar{y}$

$\overrightarrow{MG} = 2\bar{x} + 2\bar{y} = 2(\bar{x} + \bar{y}) = 2\overrightarrow{MH}$

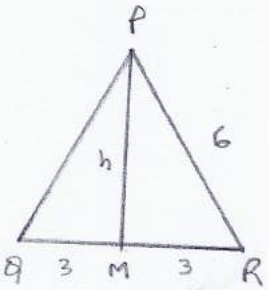
$\therefore$ 'H' lies on MG. Hence proved. [2]

10 (a) The ten circles in the diagram each have radius 1 cm.
The centre of each circle is marked with a dot.



Calculate the height of triangle *PQR*.

Rough



Consider △ PMR

Pythagoras' Theorem

$h^2 + 3^2 = 6^2$

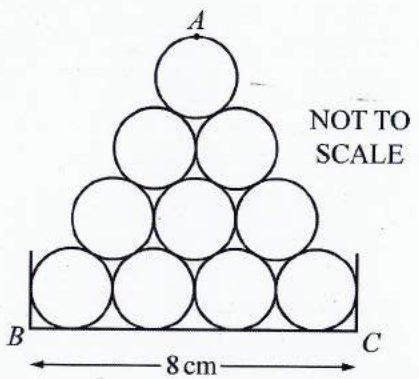
$h^2 = 36 - 9$

$h = \sqrt{27} = 5.2$ cm

5.2 ..... cm [3]

(b) Mr Patel uses whiteboard pens that are cylinders of radius 1 cm.

(i) The diagram shows 10 pens stacked in a tray.
The tray is 8 cm wide.
The point *A* is the highest point in the stack.

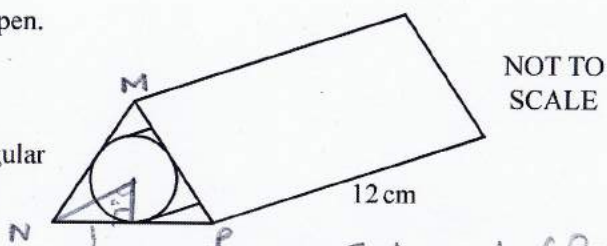


Find the height of *A* above the base, *BC*, of the tray.

Height of 'A' above 'BC' = 5.2 + 2
= 7.2 cm

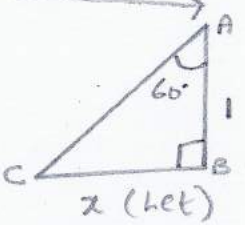
7.2 ..... cm [1]

(ii) The diagram shows a box that holds one pen.
The box is a prism of length 12 cm.
The cross section of the prism is an equilateral triangle.
The pen touches each of the three rectangular faces of the box.



Calculate the volume of this box.

Enlarged (Rough)



Step 1 Consider △ ABC

$\tan 60 = \frac{opp}{adj} = \frac{x}{1}$

$\Rightarrow x = \tan 60$

Step 2 Area of △ MNP = 6 × Area of △ ABC

$= 3 \cancel{6} \times \frac{1}{\cancel{2}} \times x \times 1$

$= 3 \times \tan 60$ ($\because x = \tan 60$)

Step 3 Volume = Area × l

$= 3 \tan 60 \times 12$

$= 36 \tan 60$

$= 62.35\ cm^3$

62.35 ..... cm^3 [5]

Question 11 is printed on the next page.

11 $f(x) = 2 - 3x$ $g(x) = 7x + 3$

(a) Find

(i) f(−3),

$f(x) = 2 - 3x$

$= 2 - 3 \times (-3)$

$= 2 + 9$

$= 11$

11 ..... [1]

(ii) g(2x).

$g(x) = 7x + 3$

$= 7(2x) + 3$

$= 14x + 3$

$14x + 3$ ..... [1]

(b) Find gf(x) in its simplest form.

$g(x) = 7x + 3$

$g[f(x)] = 7(2 - 3x) + 3$

$= 14 - 21x + 3$

$= 17 - 21x$

$17 - 21x$ ..... [2]

(c) Find x when $3f(x) = 7$.

$3f(x) = 7$

$3[2 - 3x] = 7$

$6 - 9x = 7$

$6 - 7 = 9x \Rightarrow 9x = -1$

$x = -\frac{1}{9}$

$x = -1/9$ ..... [3]

(d) Solve the equation.

$f(x + 4) - g(x) = 0$

$2 - 3(x + 4) - (7x + 3) = 0$

$2 - 3x - 12 - 7x - 3 = 0$

$-10x - 13 = 0$

$10x = -13$

$x = \frac{-13}{10} = -1.3$

$x = -1.3$ ..... [3]