LINEAR INEQUALITIES

INTRODUCTION: Two real numbers or two algebraic expressions connected by the symbol <, >, ≤ or ≥ form an inequality. It is sometimes not possible to convert statement problem into equation for example height of students in class is greater than 180cm cannot be represented in form of equation that is where importance of inequality comes.

Various types of Inequalities are:

a) Numerical Inequalities: Examples are $9>5$, $8<5$.
b) Literal Inequalities : Examples are $x\geq 4$, $y<9$
c) Double Inequalities: Examples are $2<y<9$, $1<x<15$
d) Linear Inequalities in one variables: Example is $dx+e>0$, provided $d\neq 0$.
e) Linear Inequalities in two variables: Examples is $dx+ey>f$, here two variables x and y are there.
f) Quadratic Inequalities: Example is $dx^2+ex+f>0$, it is quadratic because highest power of x is 2.

Graphical and Algebraic solution of Linear Inequation.

i) Algebraic Solution:

RULE1: From both sides of Inequality equal number can be added without affecting its sign.

RULE2: Both the sides of inequality can be divided or multiplied by the same positive number without changing sign of Inequality. But when both sides are multiplied or divided by negative number then the sign of inequality is reversed.

Example1:
Solve: $6x+3>15$
or $6x>15-3$
or $6x>12$
or $x>2$
Hence solution set is $(2,\infty)$

Example2:
Solve: $-6x+4\geq 40$
or $-6x\geq 40-4$
or $-6x\geq 36$
or $x\leq -6$ [note here we divide by negative so sign of inequality changes]

Hence solution set is $(-\infty,-6]$

Example3:
Solve: $3(2+x) \geq 4(3-x)$
or $6+2x \geq 12-4x$
or $2x+4x \geq 12-6$
or $6x \geq 6$
or $x \geq 1$
Hence solution set is $[1,\infty)$

Example4:
Solve: $4x+3 < 5x+8$
or $4x-5x < 8-3$
or $-x < 5$
$x > -5$(Dividing by negative sign changes sign)
Hence solution set is $(-5,\infty)$

Example5:
Solve: $4(1-x) \geq 12$ inequality .
$4(1-x) \geq 12$
Or $4-4x \geq 12$
Or $-4x \geq 12-4$
Or $-4x \geq 8$
Or $x \leq -2$(Dividing by negative sign changes sign)
Hence solution set is $(-\infty,-2]$

Example 6:
Solve: $4x < 12$ where x is integer
Or $x < 3$
Solution is-------- -3,-2,-1,0,1,2

a. Graphical solution of Linear Inequalities: Half plane is obtained by a line dividing Cartesian planes in two halves.
b. To find that half plane satisfies inequality, take a point (e,f) not on line and check whether it satisfies inequality or not .If it satisfies then the inequality represent that half plane and shade that region .
c. If the inequality is of type $cx+dy \geq e$ or $ex+fy \leq l$ then dark line is in the solution .

Example7:

Solve: 4x+2y≥16 graphically

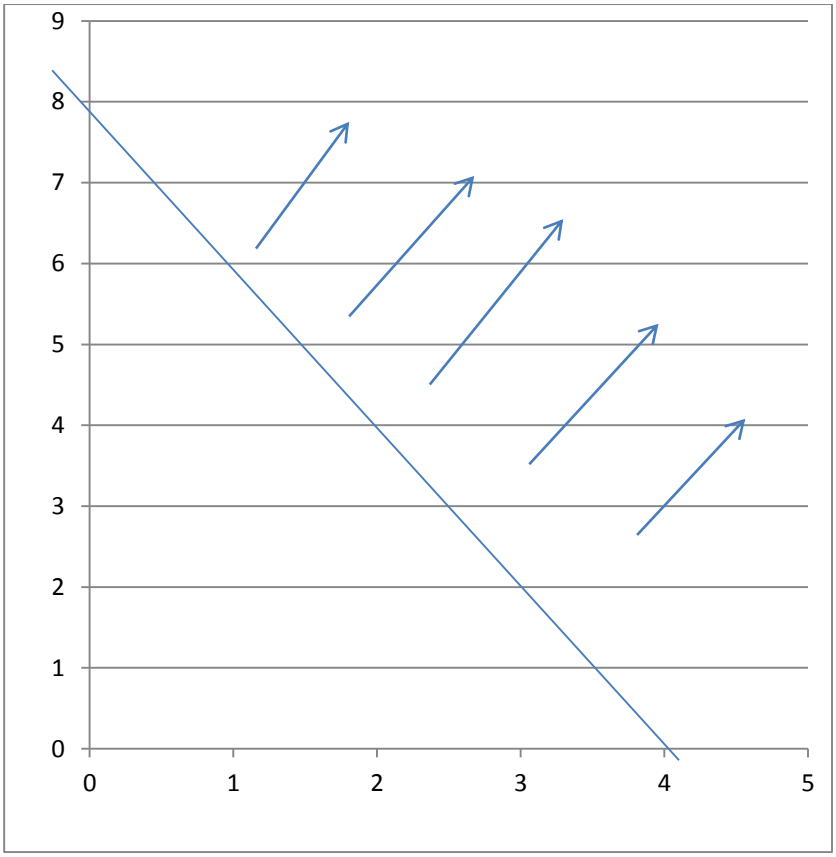
Solution: Graph 4x+2y =16 is given as dark line. This line divides xy-plane in two half planes I and II. We select a point say (0,0)and determine if this point satisfies the given inequality, we note that

4(0)+2(0) ≥16

Or 0≥16, which is false.

Hence, half plane I is not a solution region of the given inequality ,so the shaded half plane II is the solution region of inequality.

Yaxis II



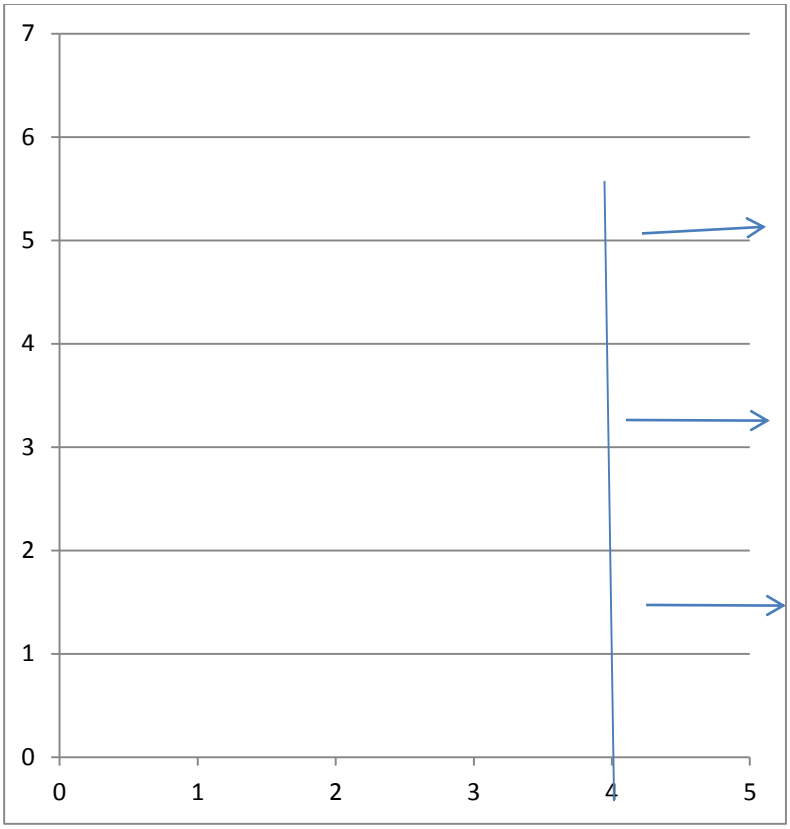
I

X axis

Example8:

Solve: 4x-16 ≥ 0 graphically

Solution: Graph of 4x-16=0 is given as dark line. This line divides xy-plane in two planes. We select a point say (0,0)and substituting it given inequality we see that 4(0)-16≥0 or -16≥ 0 which is false .Thus the solution region is the shaded region on the right hand side of a line x=4.



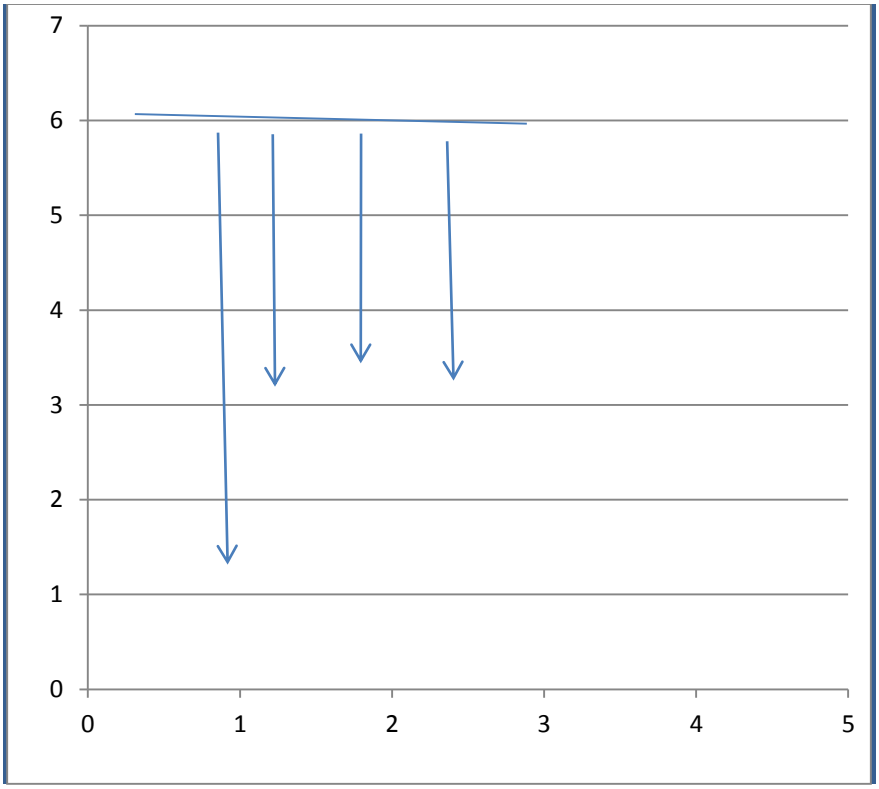
X axis

Example 9: Solve $y \leq 6$ graphically.

Solution: Graph of $y=6$ is given as dark line.

Let us select a point (0,0) and putting $y=0$ in the given inequality we see that $0<6$ which is true

Thus the solution region is the shaded region below the line $y=3$, determines the solution of the given inequality.



X axis

MCQ Inequality(Choose the Correct answer)

Q1) Which of the following is an Inequality?

a)y=5 b)y<8 c)y=17 d)y=3

Q2) what is the symbol of greater than?

a)< b)> c)= d)%

Q3) what is the direction of arrow on number line for x>5?

a)Right side towards positive infinity b) Left side towards Negative infinity c) Right Pointing towards 5 d)None.

Q4) Which of the following inequality is same as 4x + 6 < 18 ?

a)x>3 b)x<3 c)x=1 d)None

Q5) What is the solution of the inequality x+4>12 ?

a)(-∞,8) b)(8, ∞) c)(3,11) d)(-5,10)

Q6) What is the solution of the inequality 4(x+6)>0 ?

a)(- ∞,4) b)(3,9) c)(2,11) d)(-6, ∞)

Q7) What is the solution of the inequality 4(x+6)>28?

a)(1, ∞) b)(6,9) c)(8,16) d)None

Explanation and Answers

A1) b x<3 is the correct Answer as in the Inequality < sign comes.

A2)b

A3)a Right side towards positive infinity

A4)b Solid

A5)b

A6)d

A7)a

Unsolved QUESTIONS:

Solve the following Inequalities Algebraically

I. 10(3-x)≥12(1-x)

II. $\frac{x}{3}+\frac{x}{6}>9$

III. $\frac{x}{8}-\frac{x}{6}>16$

IV. 2x +14>18

V. -8x+10<18

Solve the following Inequalities by Graphical Method

a. 2x+3y≥16

b. 7x+y≥14

c. 2x+6y ≤14

d. x < -6

SUMMARY OF CHAPTER

Two real number or two algebraic expression connected by the symbol <,>,≥,≤ form an inequality.

Equal numbers may be added both sides of inequality.

a. Sign of inequality does not change when Inequality is multiplied by the same positive number .But when both sides are multiplied by a negative number then inequality is reversed.
b. Solutions of the inequality represent value of x for which Inequality is true.
c. To represent y>c and y< c on a number line, put a circle on the number c and dark line to the right (or left) of the number c.
d. For representing x≤c (or x≥c) on a number line, put a dark circle on the number c and dark the line to the left of number x or to the right as the case may be.
e. If an inequality is having ≤ or ≥ then the solutions of the inequality and the graph of inequality lies left or right of the graph of the equality represented by dark line.
f. If an inequality is a having > or < symbol, the graph of the inequality lies to the right/above or left (below) of the graph of the corresponding equality represented by dotted line
g. The solution region of a system of inequalities is the region which is satisfied by all the given inequalities in the system simultaneously.